Full-edged Real-Time Indexing for Constant Size Alphabets

Gregory Kucherov
CNRS/LIGM Marne-la-Vallée, France

Yakov Nekrich
University of Kansas, USA

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History of the Problem and Related Work

String Matching and Indexing

find all occurrences of a pattern P in a text T

- string matching : P is fixed (or given first)
- indexing : T is fixed (or given first)

Real-time string matching : early results

- for a fixed P, string matching can be done in real time independently on the alphabet size [Knuth-Morris-Pratt 77] (see also [Matiyasevich 71 (in russian)])
- language $\{P\#T : P \text{ occurs in } T\}$ can be recognized **in real time** by a Turing machine [Galil, JACM 81]

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- language $\{T\#P : P \text{ occurs in } T\}$ cannot be recognized **in real time** by (multi-tape) TM [Freidzon 68 (in russian)]
- ... but this can be done by a TM with *two-dimensional* tape if T and P are submitted on two independent input tapes [Matiyasevich 71]

Indexing under RAM model

- $\{T\#P : P \text{ occurs in } T\}$ can be recognized in real time on RAM [Slisenko 76-78]
- same result in [Kosaraju STOC 94]
- there is an index of T that can be updated in real time such that for any pattern query P made at any moment, one can check of P occurs in current T in time $O(|P|)$ [Amir,Nor SODA 08]. The result assumes a constant-size alphabet.

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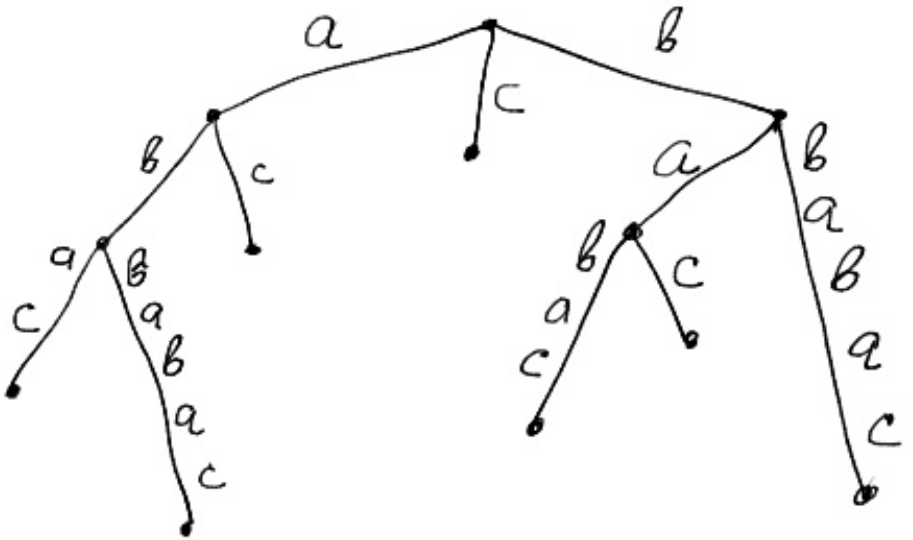
Our result : an index that can be updated in real time and all occurrences of P in the current text are reported in time $O(|P| + nb\_occ)$. The result assumes a constant-size alphabet.

Suffix Tree

Suffix Tree

abbabac



Suffix Tree

Three classical linear-time algorithms for constructing a suffix tree

- [Weiner 73] : right-to-left construction
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McCreight and Ukkonen use *suffix links* :

$$\text{for every node } v,\ v = au,\ \text{define suffix link } \mathcal{S}(v) = u$$

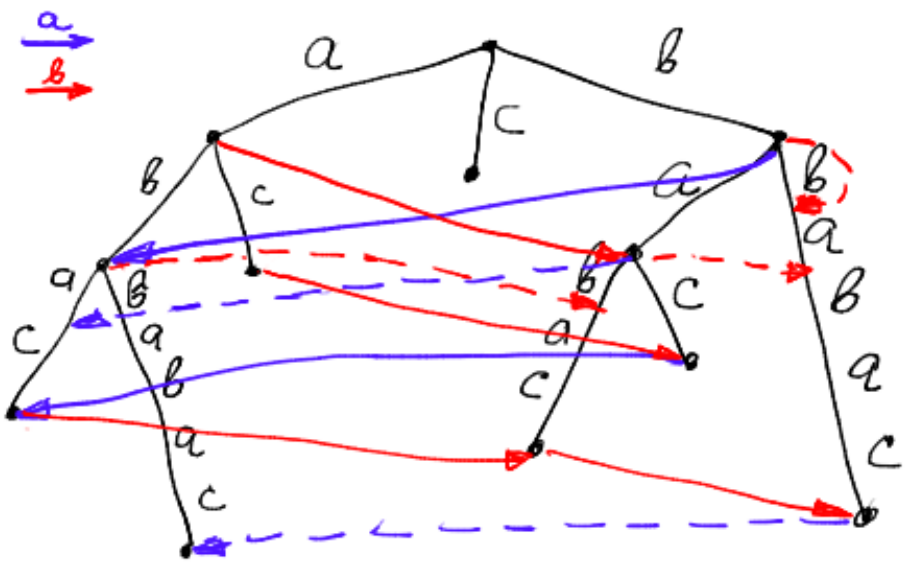
Weiner uses *prefix links* :

$$\text{for every node } v,\ \text{and for every letter } a,\ \mathcal{P}_a(v) = av$$

provided that node av exists

Prefix links

- The target of a prefix link can be an explicit or an implicit node. The prefix link is called respectively *hard* or *soft*
- If a node has a prefix link by letter a, then all its ancestors do too
- A soft link $\mathcal{P}_a(v)$ is defined iff there is a *unique* closest descendant u such that $\mathcal{P}_a(u)$ is hard, and $\mathcal{P}_a(v)$ points to edge $(w, \mathcal{P}_a(u))$



Main idea of Weiner algorithm

transforming suffix tree for t to suffix tree for at

- find the lowest ancestor u of t with a prefix link $\mathcal{P}_a(u)$
- $\mathcal{P}_a(u)$ is the branching point

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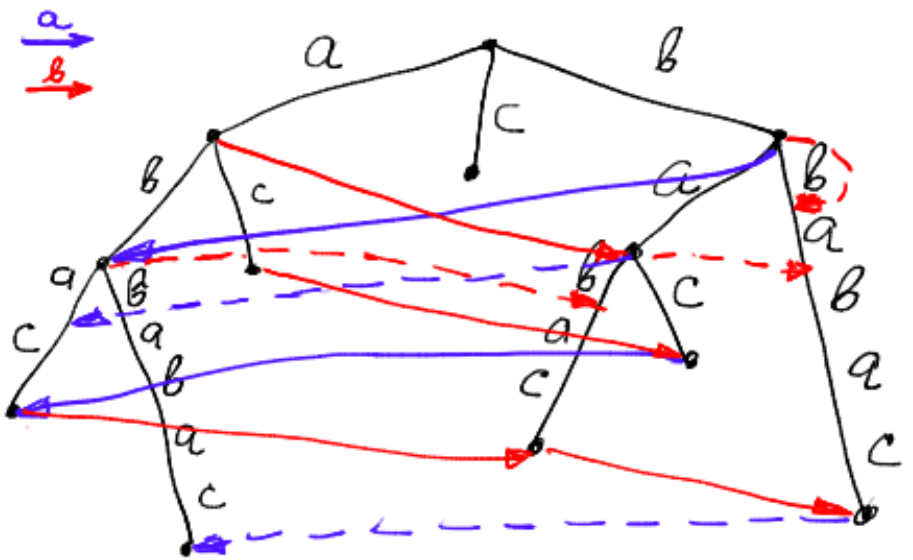

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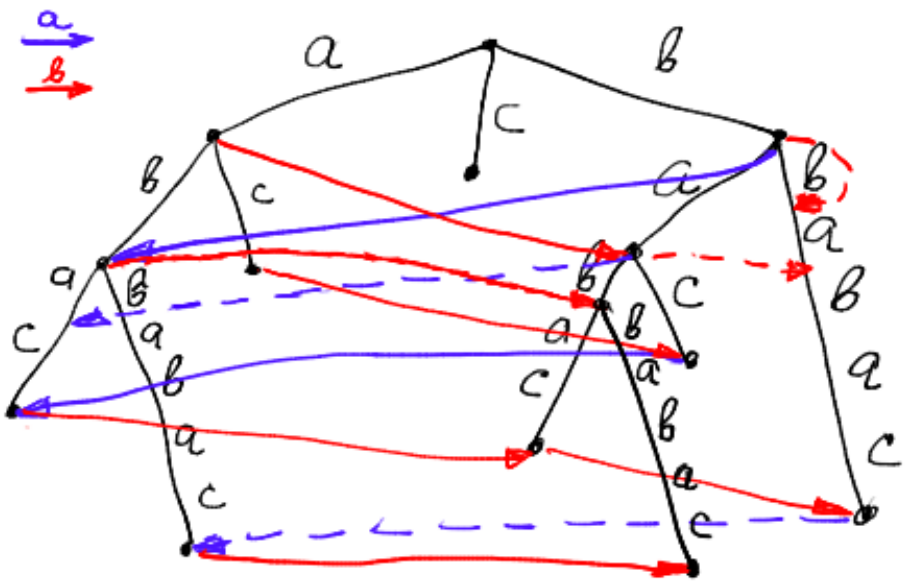

Towards real-time construction of suffix tree

- [Amir, Kopelowitz, Lewenstein, Lewenstein SPIRE 05] : $O(\log n)$ worst-case per symbol, unbounded alphabet
- [Breslauer, Italiano SPIRE 11] : $O(\log \log n)$ worst-case per symbol, constant alphabet
- [Kopelowitz FOCS 12] : $O(\log \log n + \log \log \sigma)$ *expected* worst-case per symbol, unbounded alphabet
- [Fischer, Gawrychowski arxiv 13] : $O(\log \log n + \frac{\log^2 \log \sigma}{\log \log \log \sigma})$ worst-case per symbol, unbounded alphabet

Our implementation of Weiner algorithm

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Main ideas :

- we store only hard prefix links, soft links are computed "on the fly"
- we maintain a list $\mathcal{L}_W$ corresponding to the Euler tour of the tree
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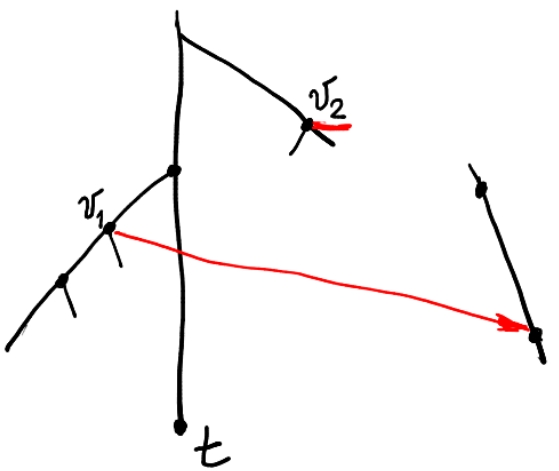

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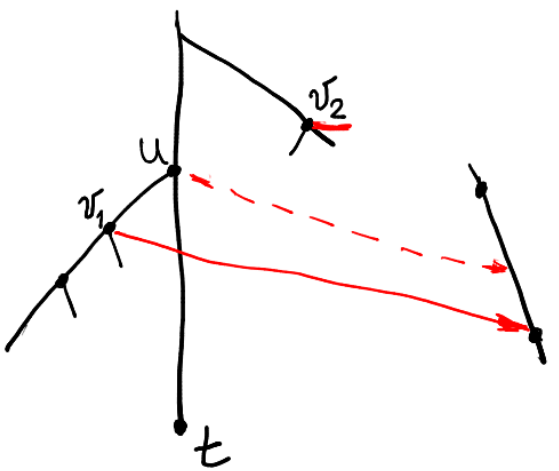

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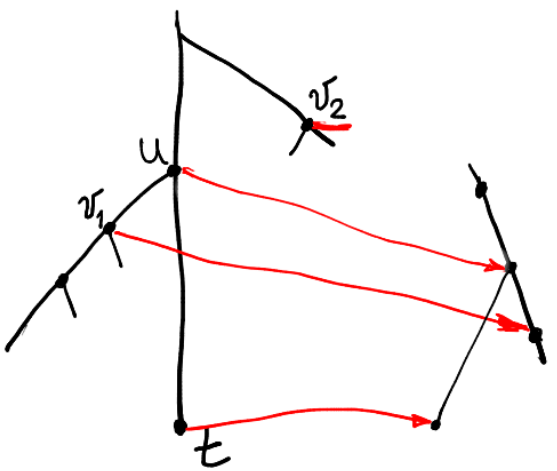

Tools that we will use

Colored Predecessor in a List

PROBLEM : Maintain a dynamic list $\mathcal{L}$ whose elements are assigned "colors". Support queries : given an element $e \in \mathcal{L}$ and a color *col*, retrieve the closest element $e' \in \mathcal{L}$ preceding e with color *col*.

THEOREM [Mortensen SODA 03, Giyora, Kaplan 09] : If the number of colors is smaller than $\log^{1/4} n$, then there exists a $O(|\mathcal{L}|)$ data structure that answers colored predecessor queries in $O(\log\log|\mathcal{L}|)$ time and supports updates (insertions) in $O(\log\log|\mathcal{L}|)$ time

Tools that we will use (cont.)

Dynamic Lowest Common Ancestor (LCA)

PROBLEM : Maintain a dynamic tree (leave insertion/deletion, edge split, edge merge) supporting lowest common ancestor of two nodes

THEOREM [Cole, Hariharan 05] : both updates and queries can be supported in worst-case $O(1)$ time

What we obtained so far

Theorem

We can maintain a suffix tree of right-to-left streaming text by spending $O(\log \log n)$ worst-case time on each symbol, assuming an alphabet size $\leq \log^{1/4} n$.

Simplifies and (slightly) generalizes [Breslauer, Italiano 11]

Our solution to real-time text indexing

Fully real-time text indexing on constant-size alphabet

Main idea :

Maintain three distinct data structures for patterns of length

- $\geq \log^2 \log n$ (long patterns),
- between $\log^2 \log \log n$ and $\log^2 \log n$ (medium-size patterns),
- $\leq \log^2 \log \log n$ (small patterns)

Data structure for long patterns (sketch)

- Group text symbols into meta-symbols of size $d = \log\log n/(4\log\sigma)$. There are $\sigma^d = \log^{1/4} n$ meta-symbols.

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- To match a long pattern P, consider all offsets δ, $0 \leq \delta \leq d-1$. For each δ, P can be matched in time $|P|/d + \log\log n$ (details left out).
- Overall we obtain time $O(d(|P|/d + \log\log n) + nb\_occ) = O(|P| + nb\_occ)$ as $|P| \geq \log^2 \log n$

Data structure for midium-size patters (sketch)

- Group text symbols into meta-symbols of size $d = \log\log\log n$. Maintain compacted trie of *truncated suffixes* of length $\log^2 \log n$ considered over the alphabet of meta-symbols
- Number of suffixes (trie leaves) is $O(d^{\log^2 \log n})$
- To maintain this trie, use (basically) the same algorithm as above. Spend $O(\log\log(d^{\log^2 \log n})) = O(\log\log\log n)$ time on each truncated suffix (i.e. amortized $O(1)$ time on each letter)
- Matching a medium-size pattern P is done similarly. The overall time is $O(d(|P|/d + \log\log\log n) + nb\_occ) = O(|P| + nb\_occ)$ as $|P| \geq \log^2 \log\log n$

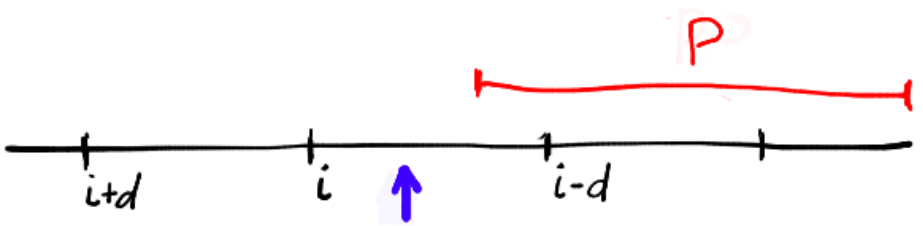
Data structure for small patterns (idea)

- Maintain a tree of truncated suffixes of length $\log^2 \log \log n$ and a list of occurrences of each truncated suffix in the current text
- Tabulate all possible trees and all possible updates
- Every update takes $O(1)$ time and a matching query takes time $O(|P| + nb\_occ)$

Turning it fully real-time

Two more problems should be overcome to make this solution real-time

Problem 1 : Most recent blocks should have a special treatment.



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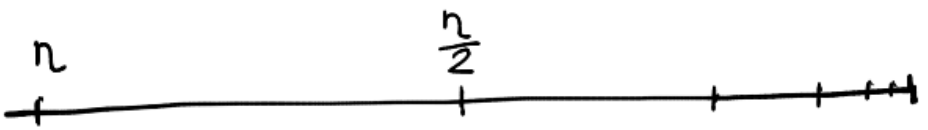
Problem 2 : Text length n is unknown.



Summary : Main result

For a streaming text over a constant-size alphabet, there exists a data structure that can be updated in real time such that at any moment, all positions of any pattern P in the current text can be reported in time $O(|P| + nb\_occ)$.